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MTH 530, Abstract Algebra I (graduate) Fall 2012 , Exam number one

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QUESTION 1. Let x and y be elements in a group G such that $xy \in Z(G)$. Prove that $xy = yx$.

QUESTION 2. a) Let G be a group such that each non identity element of G has prime order. If $Z(G) \neq \{e\}$, then prove that every non identity element of G has the same order and hence G must be group-isomorphic to Z_p for some prime p.

Find an example of a non-abelian group G where each nonidentity element has prime order.

QUESTION 3. Let $H = \{x \in U(2012) \mid 5 \mid (x-1)\}$. Prove that H is a subgroup of $U(2012)$.

QUESTION 4. Let D be a group of order q^2 for some prime number q. Prove that D must be an abelian group?

QUESTION 5. Prove that A_4 does not have a subgroup of order 6.

QUESTION 6. Let G be a group containing more than 8 elements of order 20. Prove that G is never cyclic.

QUESTION 7. Let $\alpha = (1\ 2\ 3)0(1\ 2\ 5\ 6) \in S_6$. Find $|\alpha|$ and α^{35}

QUESTION 8. Assume $|D| = 55$ and D has exactly one subgroup of order 5. Prove that D must be a cyclic group.

QUESTION 9. Let M, F be distinct proper subgroups of a group D such that D/F is group-isomorphic to D/F. Can we conclude that M is isomorphic to F? If yes, then prove it. If not, then give me a counter example.

QUESTION 10. Let D be a group of order $n > 1$ and m be a positive integer such that $gcd(n, m) = 1$. Let $b \in D$, show that there exists a unique element $f \in D$ such that $f^m = b$.

Faculty information

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